Effects of frequency-gap on explosive synchronization

Condensed Matter Theory I 02112037 Toshiki Nagasaki

1 Introduction

Synchronization is one of the central phenomena representing the emergence of collective behavior in natural and synthetic complex systems. The most successful attempt is due to Kuramoto [1]. It consists of N coupled phase oscillators $\theta_i(t)$ having natural frequencies ω_i distributed with a given probability density $g(\omega)$, and whose dynamics are governed by

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j=1}^{N} \sin(\theta_j - \theta_i) \quad ; i = 1, \cdots, N \tag{1}$$

The parameter λ accounts for the strength of the coupling among interconnected oscillators. When λ is larger than a critical value, the system is synchronized. We measure a state of the system using with the following order parameter.

$$r(t) = \frac{1}{N} \left| \sum_{i=1}^{N} e^{i\theta_i(t)} \right| \tag{2}$$

When the value reaches $r = 1$, the system is fully synchronized, while $r = 0$ for the incoherent solution. Typically, the average value of r as a function of λ displays a second-order phase transition from $r = 0$ to $r > 0$.

The studies of synchronization transition in complex networks have attracted lots of attention since the report of an abrupt percolation transition on networks, in which abrupt synchronization transitions have been reported. This phenomenon is called as an explosive synchronization transition, refers to a first-order transition where all nodes in a network abruptly get to the synchronous state with hysteresis loop when the coupling strength is larger than a critical value.

Gómez – Gardeñes et al. [2] and many researchers suggest that when the local dynamics (natural frequency) are positively correlated to the local heterogeneous structure, an explosive synchronization has been shown. Moreover, the large frequency mismatch of a pair of oscillators and the disassortativity in node degree also cause an explosive synchronization [3]. However, whether these conditions are enough to induce an explosive synchronization and which condition is the most important criterion for the emergence of an explosive synchronization transition are still open questions.

2 Models

First, We prepare a network only taking in frequency gap and observe the behavior of the synchronization transitions, varying coupling strength λ. All nodes connect to all nodes belonging to the other groups, but don't interact with the nodes whose belong to the same group. We compare the difference of the behavior, starting at random state (forward) and full synchronize state (backward). Then, those nodes are divided to two groups. Each group has N nodes. The nodes which belong to the same group have almost the same natural frequency. Second, we add one more group which has n nodes. Their natural frequencies are equal to the average frequency. We vary n and observe the behavior of the synchronization transitions.

3 Results

When we consider two groups, which means we only choose frequency-gap as a condition, and we compute Eqs.(1),(2). The system shows an explosive synchronization (Fig.1). In addition, we study analytically on backward. It shows good agreement with simulation results. Then, we add one more group. As increasing n, the area of hysteresis loop decreases (Fig.2). Especially, when n gets over $2N$, the area vanishes. Therefore, when n/N is small, we can observe an explosive synchronization.

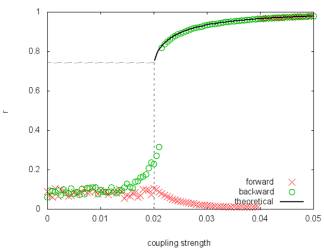

Fig. 1: Explosive synchronization with two groups ($N = 50$)

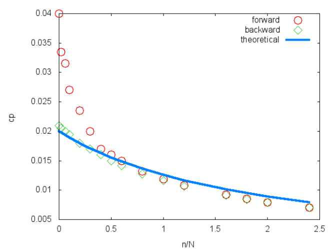

Fig. 2: n/N vs the area of hysteresis loop

4 Conclusion

To summarize, we have explored the important factors which induce an explosive synchronization with simulations and analytical calculations. In recent study, some possible factors are proposed. Especially, it is widely believed the large frequency mismatch of a pair of oscillators is a direct cause of an explosive synchronization. So, in our study, we constructed a simple model with such factor and observed a behavior of synchronization transition. As a result, we can observe an explosive synchronization when the nodes which have average frequency are relatively fewer than other nodes whose natural frequencies are not equal to the average frequency ($n/N < 2$). However, when n gets over $2N$, the hysteresis loop disappears and the transition becomes continuous. This result suggests that, as long as frequency gap exists, explosive synchronization transition occurs.

References

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